Linear Algebra I

18/12/2015, Friday, 9:00–11:00

You are **NOT** allowed to use any type of calculators.

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1 $(2+6+4+4+4=20 \text{ pts})$ **Linear equations**

Consider the following linear system of equations

$$\begin{aligned} d + e - f &= 2 \\ a + 2b + e - f &= 0 \\ a + 2b + 2c - e + f &= 2. \end{aligned}$$

(a) Write down the augmented matrix.

(b) By performing row operations, put the augmented matrix into row echelon form.

(c) Determine the *lead* and *free* variables.

(d) By performing row operations, put the augmented matrix into row *reduced* echelon form.

(e) Find the solution set of the equation.

2 $(10+10=20 \text{ pts})$ **Determinant and inverse matrix**

Consider the matrix

$$\begin{pmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{pmatrix}.$$

1. Find the determinant.
2. Determine all values of x for which this matrix is nonsingular.

3 $(10+10=20 \text{ pts})$ **Partitioned matrices**

Let A, B, and C be $n \times n$ matrices and

$$M = \begin{bmatrix} A & B \\ C & 0_{n\times n} \end{bmatrix}.$$

1. Show that M is nonsingular if and only if both B and C are nonsingular.
2. Suppose that B and C are nonsingular. Find the inverse of M.

4 $(2+4+6+6+4+8=30$ pts) **Vector spaces**

Consider the vector space P_4. Let $S=\{p(x)\in P_4 \mid p(x)+p(-x)=0\}$ and $L: P_4\to P_4$ be given by $L(p(x))=\dfrac{1}{2}(p(x)+p(-x))$.

1. Are the vectors $1+x$, $x+x^2$, x^2+x^3, x^3+1 linearly independent?
2. Are the vectors $1+x$, $x+x^2$, x^2+x^3, x^3 for a basis for P_4?
3. Show that the set S is a subpace of P_4. Find a basis for S and determine its dimension.
4. Show that L is a linear transformation.
5. Find $\ker L$.
6. Find the matrix representation of L with respect to the ordered basis $\{1+x,\ x+x^2,\ x^2+x^3,\ x^3\}$

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10 pts free